AN INVESTIGATION ON EVALUATION METHOD FOR RELIABILITY OF THREE DIMENSIONAL SUBSURFACE GEOLOGICAL MODEL

Shinji Masumoto[1], Tatsuya Nemoto[1], Venkatesh Raghavan[2] and Susumu Nonogaki[3]

[1]Department of Geosciences, Graduate School of Science, Osaka City University,
3-3-138 Sugimoto, Sumiyoshi-ku, Osaka 558-8585, Japan
Email: masumoto@sci.osaka-cu.ac.jp
Email: tnemoto@sci.osaka-cu.ac.jp
[2]Graduate School for Creative Cities, Osaka City University,
3-3-138 Sugimoto, Sumiyoshi-ku, Osaka 558-8585, Japan
Email: raghavan@media.osaka-cu.ac.jp
[3]National Institute of Advanced Industrial Science and Technology, Japan,
Central 7, 1-1-1 Higashi, Tsukuba, Ibaraki 305-8567, Japan
Email: s-nonogaki@aist.go.jp

ABSTRACT

The reliability of the three dimensional subsurface geological model affects the result of the geological analysis. The consistency of the geological model using geostatistical method has been discussed as regards the uncertainty. However, the reliability of the geological models using spline estimation method for geologic boundary surface cannot be derived directly. In this study, we examined the expression method for the reliability of the spline geological surface model based on the density of the data and a new evaluation function of the surface estimation method.

1. INTRODUCTION

Recently, there are many problems such as environmental pollution and mitigation/prevention of natural disaster which should be carefully considered using geologic information. As one of the solutions of this problem, it is necessary to provide the geologic information accurately and effectively by the use of a 3D geologic model. In the construction of the 3D geologic model, optimized spline interpolation method has come to be used frequently for the estimation of the geologic boundary surface using borehole data and field survey data (*e.g.* Kimura *et al.*, 2013). It is necessary to express the reliability of the model, because the reliability of the geologic model relates the quality of the geologic analysis. The reliability of the geological model created by the geostatistical method has been discussed in terms of uncertainty (*e.g.* Tacher *et al.*, 2006). However, the reliability of the geological models using spline estimation method for geologic boundary surface cannot be calculated directly. The reason being that spline estimation method can create the surface to approximately satisfy all of the data.

In order to solve this problem, a few evaluation methods for the reliability of 3D geologic model using data density have been discussed (*e.g.* Masumoto *et al.*, 2012). In this study, a new evaluation method for the reliability of boundary surface based on the kernel density estimation using variation of surface shape in addition to data density has been developed.

2. BASIC THEORY FOR RELIABILITY EVALUATION

In general a reliability of 3D geologic model increases when boundary surface are estimated by appropriate data set. In case of surface estimation, it will, obviously, be better when there will be more data. In addition, for the part of high variation of surface, high density data is necessary. Therefore, the evaluation method for reliability based on the data density corrected by the variation of surface shape are examined.

2.1 Data Density Estimation

To obtain data density for surface estimation, the kernel density estimation method has been extended. Kernel density estimation is a non-parametric method, defined by the following equation in one-dimension.

$$\hat{f}_h(x) = \sum_{i=1}^{n} K_h(x - x_i) = \frac{1}{nh}\sum_{i=1}^{n} K(\frac{x - x_i}{h}),$$

where, $K(u)$ is the kernel function, n is a number of data points, h is a band width and $x - x_i$ is distance between data point x_i and calculation point x.

There are various expressions for the kernel function, such as Triangular, Gaussian and Epanechnikov using the following equation (Figure 1(a)).

Triangular $$K(u) = (1 - |u|)1_{\{|u| \le 1\}},$$

Gaussian $$K(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2},$$

Epanechnikov $$K(u) = \frac{3}{4}(1 - u^2)1_{\{|u| \le 1\}}.$$

In above equations, $1_{\{|u| \le 1\}}$ is the indicator function.

In the case of 2 dimension, multivariate kernel density estimation has been used. The examples of kernel function for surface estimation are shown in Figure 1(b), (c) and (d). In order to estimate the geologic boundary surface, equality and inequality elevation data are mainly used (Figure 2). Reliability of inequality elevation data is relatively small compared to the equality elevation data. According to this logic, different weight for the kernel density estimation has been used. The weight of data can be defined by the height of kernel function.

$$\hat{f}_h(x) = \frac{1}{h\sum_{i=1}^{n} w_i}\sum_{i=1}^{n}(w_i K(\frac{x - x_i}{h})),$$

where w_i is weight for height of kernel function (Figure 3(a) and (b)).

Corresponding to the data type, equality data are $w_i = 1$, and inequality data are $w_i = p_1$ ($0 < p_1 < 1$). The distribution of data density calculated by above show two important properties. The values of the density indicate the quantity of data used for surface estimation. Secondly, the titling of density surface is related to the degree indicating the extent to which data is scattered. A steep slope shows uneven and inclined distribution to a certain direction of the data point. Therefore, high density and low slope suggest high reliability of surfaces.

2.2 Variation of surface shape

A true surface of 3D geologic model is unknown. For calculation of a variation of shape of surface estimated by the existing data at this time has been used. There are many

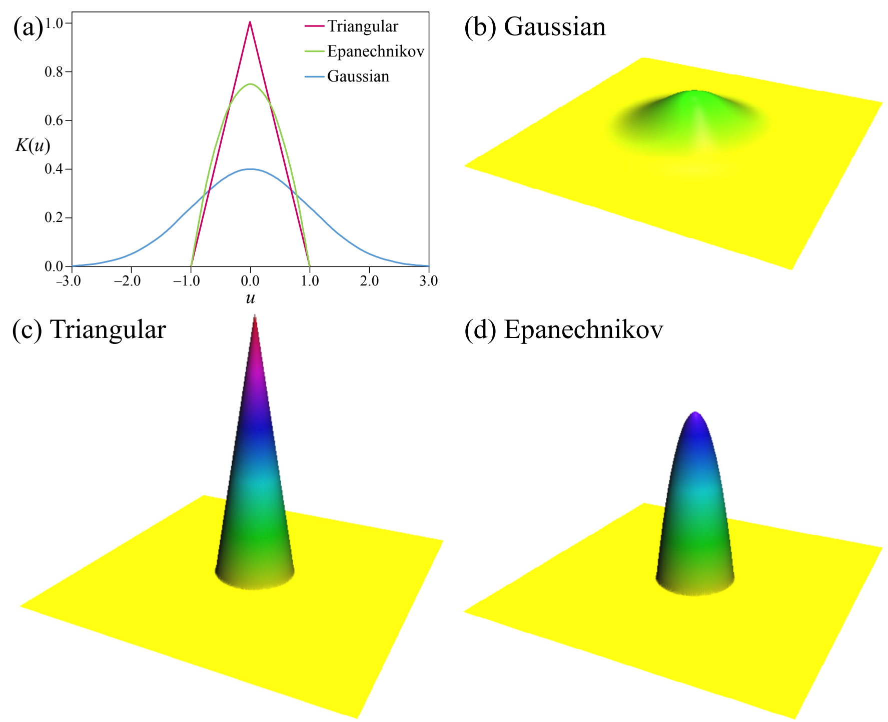

Figure 1. Examples of kernel function for surface estimation

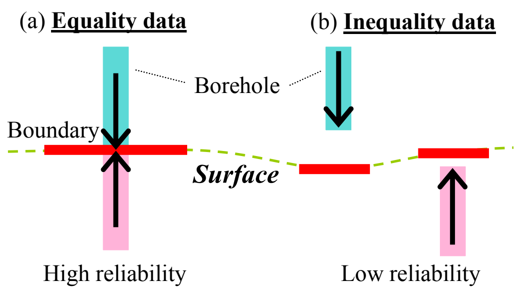

Figure 2. Equality and inequality elevation data

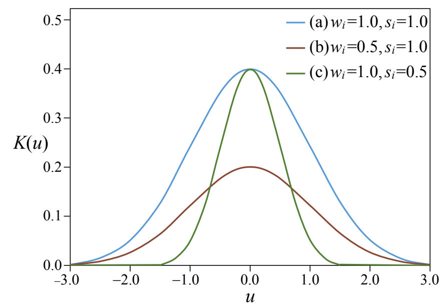

Figure 3. Variation of kernel function

methods to evaluate the shape of the surface. For example, first-order or second-order differentiation are often adopted. The first-order differentiation cannot be used because geologic boundary surface does not need to be horizontal. The second-order differentiation which shows flatness of surface is appropriate to the evaluation of surface variation. Therefore, the second-order differentiation has been used. In this study, the second-order differentiation defined by the following functional equation $J(f)$ which is in the same form as evaluation equation for surface estimation has been used for variation of surface f (Shiono *et al.*, 2001).

$$J(f) = \int\int_{\Omega} \{(\frac{\partial^2 f}{\partial x^2})^2 + (\frac{\partial^2 f}{\partial x \partial y})^2 + (\frac{\partial^2 f}{\partial y^2})^2\} dxdy$$

For discrete data, $J(f)$ can be approximately expressed by the following formula.

$$J(f)=\sum_{j=1}^{Ny}\sum_{i=2}^{Nx-1}(f_{i+1,j}-2f_{i,j}+f_{i-1,j})^2+2\sum_{i=1}^{Ny-1}\sum_{i=1}^{Nx-1}(f_{i,j}-f_{i+1,j}-2f_{i,j+1}-f_{i+1,j+1})^2+\sum_{j=2}^{Ny-1}\sum_{i=1}^{Nx}(f_{i,j+1}-2f_{i,j}+f_{i,j-1})^2$$

2.3 Reliability of Surface

The variation of surface is reflected by calculation of the data density. In the location of large variation of surface, it can be assumed that the effective region of each data is small. According to this logic, the band width of the kernel function is set to a small value in the large variation area of the surface. Thus, new variable s_i for the control of band width has been used. Finally, the reliability of surface can be defined by the following equation.

$$\hat{f}_h(x)=\frac{1}{h\sum_{i=1}^{n}w_i}\sum_{i=1}^{n}(w_i\,K(\frac{x-x_i}{h\,s_i}))$$

Corresponding to the variation of surface, variable s_i is $s_i = 1$ for the no variation area, and s_i is $s_i = p_2$ $(0 < p_2 < 1)$ for the large variation area (Fig. 3(c)). However, the parameter p_1 and p_2 have not been determined at the present and more detailed research is necessary.

3. EXAMPLE OF RELIABILITY

As an example, the reliability of surface have been calculated using test model with large variation of surface. The example of test model surface and sampling data points are

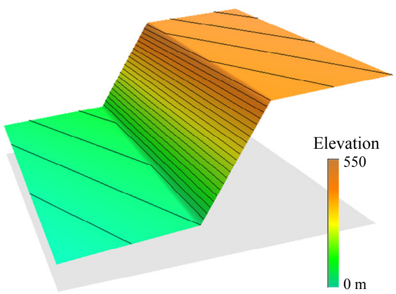

Figure 4. Test model surface

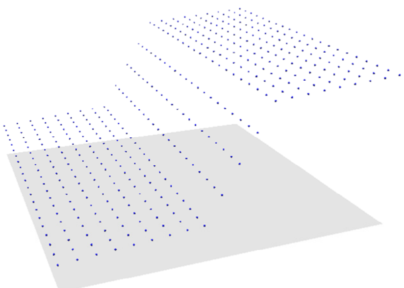

Figure 5. Data sampling points

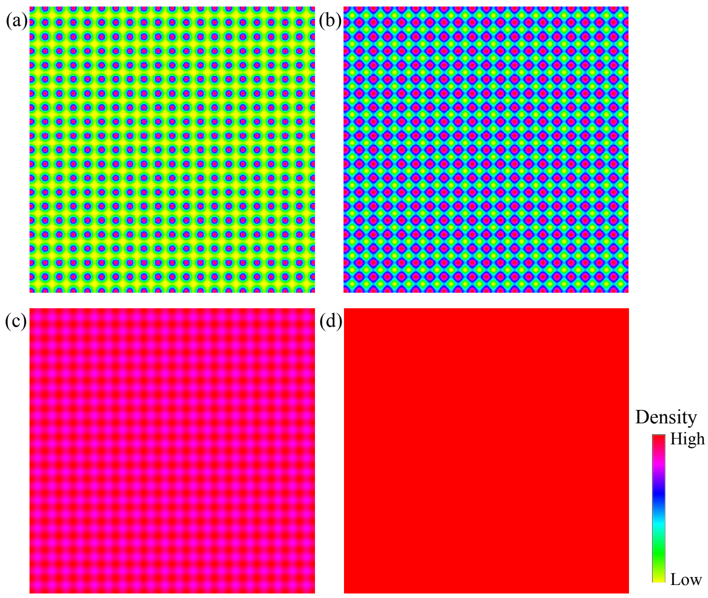

Figure 6. Results of kernel density estimation (a) *h* = 10, (b) *h* = 20, (c) *h* = 25 and (d) *h* = 40.

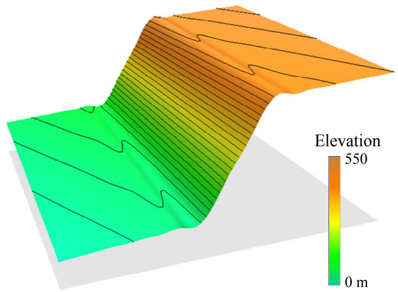

Figure 7. Estimated surface

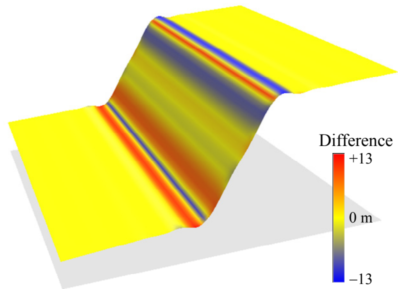

Figure 8. Difference between test model surface and estimated surface

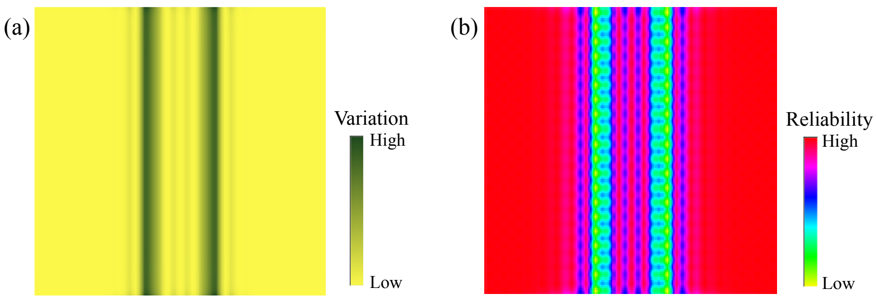

Figure 9. Results of calculation (a) Variation and (b) reliability.

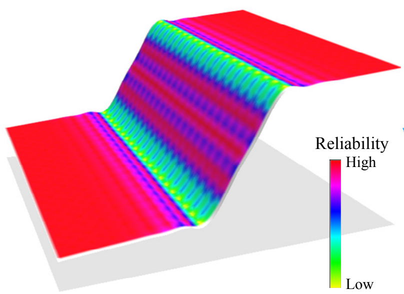

Figure 10. Reliability of surface

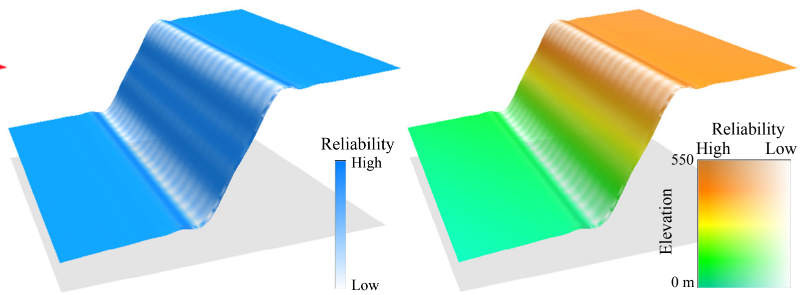

Figure 11. Demonstrations of reliability using transparency of surface

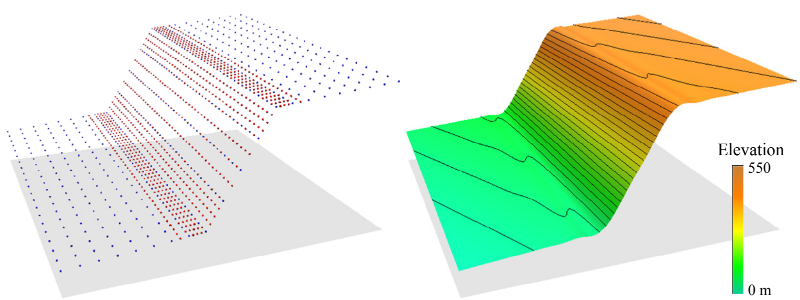

Figure 12. Resampling data points and recalculated surface

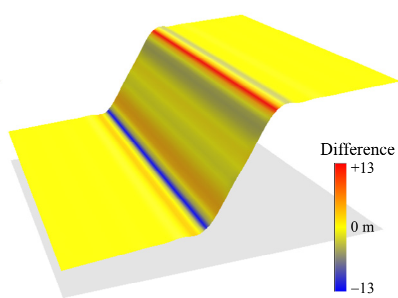

Figure 13. Difference between test model surface and recalculated surface

shown in Figure 4 and 5. The results of kernel density estimation using Gaussian kernel function are shown in Figure 6. It can be inferred from these results that if band width is small then density is high around data point. Figure 7 shows the estimated surface using BS-Horizon program (Nonogaki *et al*., 2008). Difference between test model surface and estimated surface is shown in Figure 8. There are huge gaps from test model such as overshoot or undershoot occurring in the estimated surface. Figure 9 and 10 show the variation and the reliability using variation of surface based on the basic theory. In Figure 11, the reliability using transparency of surface have been demonstrated. In order to confirm, the surface has been recalculated using additional data appended high density data for the low reliability area (Figure 12). Difference between test model surface and recalculated surface is shown in Figure 13. In comparison with Figure 8, the difference of surfaces is seen to have reduced remarkably.

4. CONCLUSIONS

The evaluation method for the reliability of geologic boundary surface has been developed using kernel density estimation. The reliability estimate has many applications such as survey route planning for reconstruction of 3D geologic model and connecting two adjacent 3D models created individually in addition to the original purpose of 3D geological model generation. In case of practical application, further development and improvement of this reliability expression are necessary.

This study was supported by KAKENHI (25330134; Grant-in-Aid for Scientific Research by Japan Society for Promotion of Science).

REFERENCES

Kimura K., Masumoto S., Takano O. and Nemoto T., 2013. Recent progress in the three-dimensional geological modeling. *Jour. Geol. Soc. Japan*, 119(8), 509-514.

Masumoto S., Nemoto T., Nonogaki S., Tawara H. and Raghavan V., 2012. A study of expression method for reliability of three dimensional geologic model. *Proc. Intern. Symp. GeoInformatics for Spatial-Infrastructure Development in Earth and Allied Sciences* (*GIS-IDEAS*) *2008*, 83-88.

Nonogaki, S., Masumoto S. and Shiono K., 2008. Optimal determination of geologic boundary surface using cubic B-spline. *Geoinformatics*, 19(2), 61-77.

Shiono K., Noumi Y., Masumoto S. and Sakamoto M., 2001. Horizon2000 : Revised Fortran program for optimal determination of geologic surfaces based on field observation including equality-inequality constraints and slop information. *Geoinformatics*, 12(4), 229-249.

Tacher, L., Pomian-Srzednicki, I. and Parriaux, A., 2006. Geological uncertainties associated with 3-D subsurface models. *Computers & Geosciences*, 32(2), 212–221.